1.

(a) $f(1) = e^2$

$f'(x) = 2e^{2x} \Rightarrow f'(1) = 2e^2$

An equation for the tangent line is $y = e^2 + 2e^2(x-1)$.

(b) Area $= \int_0^1 e^{2x}\,dx = \left[\frac{1}{2}e^{2x}\right]_{x=0}^{x=1} = \frac{1}{2}\left(e^2 - 1\right)$

(c) Volume $= 1 + \int_1^{e^2} \left(1 - \frac{1}{2}\ln y\right)^2 dy$

2.

(a) $x'(t) = 15t^2 - 18t$

$x'(1) = 15 - 18 = -3$

Since $x'(1) < 0$, the particle is moving to the left at time $t = 1$.

(b) $x'(t) = 3t(5t - 6) = 0 \Rightarrow t = 0,\ t = \frac{6}{5}$

Since $x'(t) < 0$ for $0 < t < \frac{6}{5}$ and $x'(t) > 0$ for $t > \frac{6}{5}$,

the particle is farthest to the left at time $t = \frac{6}{5}$.

(c) Area $= A(t) = \frac{1}{2}x(t)y(t)$

$$= \frac{1}{2}\left(5t^3 - 9t^2 + 7\right)(7t + 3)$$

$$A'(t) = \frac{1}{2}\left[\left(15t^2 - 18t\right)(7t + 3) + \left(5t^3 - 9t^2 + 7\right)(7)\right]$$

$$A'(1) = \frac{1}{2}[(-3)(10) + (3)(7)] = \frac{1}{2}[-30 + 21] = -\frac{9}{2}$$