21. Functions w, x, and y are differentiable with respect to time and are related by the equation $w = x^2y$. If x is decreasing at a constant rate of 1 unit per minute and y is increasing at a constant rate of 4 units per minute, at what rate is w changing with respect to time when $x = 6$ and $y = 20$?

(A) −384   (B) −240   (C) −96   (D) 276   (E) 384

22. Let f be the function defined by $f(x) = 2x^3 - 3x^2 - 12x + 18$. On which of the following intervals is the graph of f both decreasing and concave up?

(A) $(-\infty, -1)$ (B) $\left(-1, \frac{1}{2}\right)$ (C) $(-1, 2)$ (D) $\left(\frac{1}{2}, 2\right)$ (E) $(2, \infty)$

$$f(x) = \begin{cases} 3x + 5 & \text{when } x < -1 \\ -x^2 + 3 & \text{when } x \geq -1 \end{cases}$$

23. If f is the function defined above, then $f'(-1)$ is

(A) −3   (B) −2   (C) 2   (D) 3   (E) nonexistent

24. Let f be the function defined by $f(x) = \dfrac{(3x + 8)(5 - 4x)}{(2x + 1)^2}$. Which of the following is a horizontal asymptote to the graph of f ?

(A) $y = -6$

(B) $y = -3$

(C) $y = -\dfrac{1}{2}$

(D) $y = 0$

(E) $y = \dfrac{3}{2}$

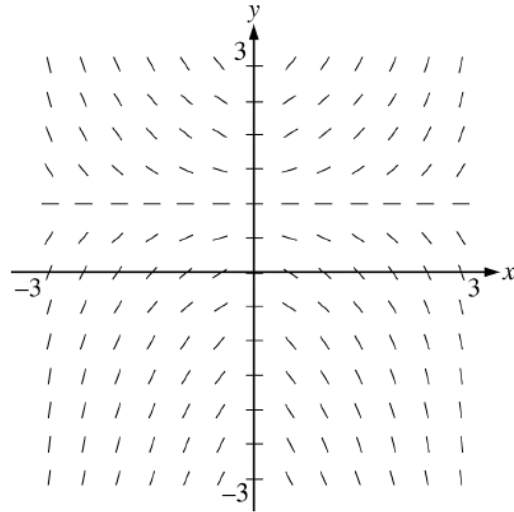25. If $y = x^2 - 2x$ and $u = 2x + 1$, then $\dfrac{dy}{du} =$

(A) $\dfrac{2(x^2 + x - 1)}{(2x + 1)^2}$ (B) $6x^2 - 3x - 2$ (C) $4x$ (D) $x - 1$ (E) $\dfrac{1}{x - 1}$

26. For $x > 0$, $\dfrac{d}{dx} \displaystyle\int_1^{\sqrt{x}} \dfrac{1}{1 + t^2}\, dt =$

(A) $\dfrac{1}{2\sqrt{x}\,(1 + x)}$ (B) $\dfrac{1}{2\sqrt{x}\,(1 + \sqrt{x})}$ (C) $\dfrac{1}{1 + x}$ (D) $\dfrac{\sqrt{x}}{1 + x}$ (E) $\dfrac{1}{1 + \sqrt{x}}$

27. A particle moves on the x-axis so that at any time t, $0 \leq t \leq 1$, its position is given by $x(t) = \sin(2\pi t) + 2\pi t$. For what value of t is the particle at rest?

(A) 0   (B) $\dfrac{1}{8}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{2}$ (E) 1

28. Shown above is a slope field for which of the following differential equations?

(A) $\dfrac{dy}{dx} = xy - x$

(B) $\dfrac{dy}{dx} = xy + x$

(C) $\dfrac{dy}{dx} = y - x^2$

(D) $\dfrac{dy}{dx} = (y - 1)x^2$

(E) $\dfrac{dy}{dx} = (y - 1)^3$

76. A particle moves along a straight line so that at time $t > 0$ the position of the particle is given by $s(t)$, the velocity is given by $v(t)$, and the acceleration is given by $a(t)$. Which of the following expressions gives the average velocity of the particle on the interval $[2, 8]$?

(A) $\dfrac{1}{6}\displaystyle\int_2^8 a(t)\,dt$

(B) $\dfrac{1}{6}\displaystyle\int_2^8 s(t)\,dt$

(C) $\dfrac{s(8) - s(2)}{6}$

(D) $\dfrac{v(8) - v(2)}{6}$

(E) $v(8) - v(2)$

78. The function f is differentiable and increasing for all real numbers x, and the graph of f has exactly one point of inflection. Of the following, which could be the graph of f', the derivative of f?

(A)

(B)

(C)

(D)

(E)