Show all work that leads to your answers!

1. Free response

Question 3

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \le 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

(a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

(c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \dfrac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

(d) Write an expression for Nathan's acceleration in terms of t.

Multiple Choice

1. $\int \left(5e^{2x} + \dfrac{1}{x}\right) dx =$

(A) $\dfrac{5}{2}e^{2x} + \dfrac{2}{x^2} + C$

(B) $\dfrac{5}{2}e^{2x} + \ln|x| + C$

(C) $5e^{2x} + \dfrac{2}{x^2} + C$

(D) $5e^{2x} + \ln|x| + C$

(E) $10e^{2x} - \dfrac{1}{x^2} + C$

2. If $f(x) = \sqrt{x} + \frac{3}{\sqrt{x}}$, then $f'(4) =$

(A) $\frac{1}{16}$ (B) $\frac{5}{16}$ (C) 1 (D) $\frac{7}{2}$ (E) $\frac{49}{4}$

3. $\int x^2 \left(x^3 + 5\right)^6 \, dx =$

(A) $\frac{1}{3}\left(x^3 + 5\right)^6 + C$

(B) $\frac{1}{3}x^3\left(\frac{1}{4}x^4 + 5x\right)^6 + C$

(C) $\frac{1}{7}\left(x^3 + 5\right)^7 + C$

(D) $\frac{3}{7}x^2\left(x^3 + 5\right)^7 + C$

(E) $\frac{1}{21}\left(x^3 + 5\right)^7 + C$

x	0	25	30	50
$f(x)$	4	6	8	12

4. The values of a continuous function f for selected values of x are given in the table above. What is the value of the left Riemann sum approximation to $\int_0^{50} f(x)\,dx$ using the subintervals $[0, 25]$, $[25, 30]$, and $[30, 50]$?

(A) 290 (B) 360 (C) 380 (D) 390 (E) 430

$$f(x) = \begin{cases} x^2 \sin(\pi x) & \text{for } x < 2 \\ x^2 + cx - 18 & \text{for } x \geq 2 \end{cases}$$

5. Let f be the function defined above, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

(A) 2 (B) 7 (C) 9 (D) $4\pi - 4$ (E) There is no such value of c.