1.

(a) $\int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$

$\Rightarrow 7 = \int_{-6}^{-2} f(x)\,dx + 2 + \left(9 - \frac{9\pi}{4}\right)$

$\Rightarrow \int_{-6}^{-2} f(x)\,dx = 7 - \left(11 - \frac{9\pi}{4}\right) = \frac{9\pi}{4} - 4$

(b) $\int_{3}^{5} (2f'(x) + 4)\,dx = 2\int_{3}^{5} f'(x)\,dx + \int_{3}^{5} 4\,dx$

$= 2(f(5) - f(3)) + 4(5 - 3)$

$= 2\left(0 - \left(3 - \sqrt{5}\right)\right) + 8$

$= 2\left(-3 + \sqrt{5}\right) + 8 = 2 + 2\sqrt{5}$

— OR —

$\int_{3}^{5} (2f'(x) + 4)\,dx = [2f(x) + 4x]_{x=3}^{x=5}$

$= (2f(5) + 20) - (2f(3) + 12)$

$= (2 \cdot 0 + 20) - \left(2\left(3 - \sqrt{5}\right) + 12\right)$

$= 2 + 2\sqrt{5}$

(c) $g'(x) = f(x) = 0 \Rightarrow x = -1,\ x = \frac{1}{2},\ x = 5$

x	$g(x)$
-2	0
-1	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{4}$
5	$11 - \frac{9\pi}{4}$

On the interval $-2 \le x \le 5$, the absolute maximum value of g is $g(5) = 11 - \frac{9\pi}{4}$.

(d) $\lim_{x \to 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} = \frac{10^1 - 3f'(1)}{f(1) - \arctan 1}$

$= \frac{10 - 3 \cdot 2}{1 - \arctan 1} = \frac{4}{1 - \frac{\pi}{4}}$

2.

(a) $V = \pi r^2 h = \pi(1)^2 h = \pi h$

$$\left.\frac{dV}{dt}\right|_{h=4} = \pi \left.\frac{dh}{dt}\right|_{h=4} = \pi\left(-\frac{1}{10}\sqrt{4}\right) = -\frac{\pi}{5} \text{ cubic feet per second}$$

(b) $$\frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} = -\frac{1}{20\sqrt{h}} \cdot \left(-\frac{1}{10}\sqrt{h}\right) = \frac{1}{200}$$

Because $\frac{d^2h}{dt^2} = \frac{1}{200} > 0$ for $h > 0$, the rate of change of the height is increasing when the height of the water is 3 feet.

(c) $$\frac{dh}{\sqrt{h}} = -\frac{1}{10}\,dt$$

$$\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{10}\,dt$$

$$2\sqrt{h} = -\frac{1}{10}t + C$$

$$2\sqrt{5} = -\frac{1}{10} \cdot 0 + C \Rightarrow C = 2\sqrt{5}$$

$$2\sqrt{h} = -\frac{1}{10}t + 2\sqrt{5}$$

$$h(t) = \left(-\frac{1}{20}t + \sqrt{5}\right)^2$$