**1.**

(a) $\int_0^2 (h(x) - g(x))\,dx = \int_0^2 \left(\left(6 - 2(x-1)^2\right) - \left(-2 + 3\cos\left(\frac{\pi}{2}x\right)\right)\right) dx$

$$= \left[\left(6x - \frac{2}{3}(x-1)^3\right) - \left(-2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right)\right)\right]_{x=0}^{x=2}$$

$$= \left(\left(12 - \frac{2}{3}\right) - (-4 + 0)\right) - \left(\left(0 + \frac{2}{3}\right) - (0 + 0)\right)$$

$$= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}$$

The area of $R$ is $\frac{44}{3}$.

(b) $\int_0^2 A(x)\,dx = \int_0^2 \frac{1}{x+3}\,dx$

$$= \left[\ln(x+3)\right]_{x=0}^{x=2} = \ln 5 - \ln 3$$

The volume of the solid is $\ln 5 - \ln 3$.

(c) $\pi \int_0^2 \left((6 - g(x))^2 - (6 - h(x))^2\right) dx$

**2.**

(a) $h'(2) = \frac{2}{3}$

(b) $a'(x) = 9x^2 h(x) + 3x^3 h'(x)$

$$a'(2) = 9 \cdot 2^2 h(2) + 3 \cdot 2^3 h'(2) = 36 \cdot 4 + 24 \cdot \frac{2}{3} = 160$$

(c) Because $h$ is differentiable, $h$ is continuous, so $\lim_{x \to 2} h(x) = h(2) = 4$.

Also, $\lim_{x \to 2} h(x) = \lim_{x \to 2} \frac{x^2 - 4}{1 - (f(x))^3}$, so $\lim_{x \to 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4$.

Because $\lim_{x \to 2} \left(x^2 - 4\right) = 0$, we must also have $\lim_{x \to 2} \left(1 - (f(x))^3\right) = 0$.

Thus $\lim_{x \to 2} f(x) = 1$.

Because $f$ is differentiable, $f$ is continuous, so $f(2) = \lim_{x \to 2} f(x) = 1$.

Also, because $f$ is twice differentiable, $f'$ is continuous, so $\lim_{x \to 2} f'(x) = f'(2)$ exists.

Using L'Hospital's Rule,

$$\lim_{x \to 2} \frac{x^2 - 4}{1 - (f(x))^3} = \lim_{x \to 2} \frac{2x}{-3(f(x))^2 f'(x)} = \frac{4}{-3(1)^2 \cdot f'(2)} = 4.$$

Thus $f'(2) = -\frac{1}{3}$.

(d) Because $g$ and $h$ are differentiable, $g$ and $h$ are continuous, so $\lim_{x \to 2} g(x) = g(2) = 4$ and $\lim_{x \to 2} h(x) = h(2) = 4$.

Because $g(x) \le k(x) \le h(x)$ for $1 < x < 3$, it follows from the squeeze theorem that $\lim_{x \to 2} k(x) = 4$.

Also, $4 = g(2) \le k(2) \le h(2) = 4$, so $k(2) = 4$.

Thus $k$ is continuous at $x = 2$.