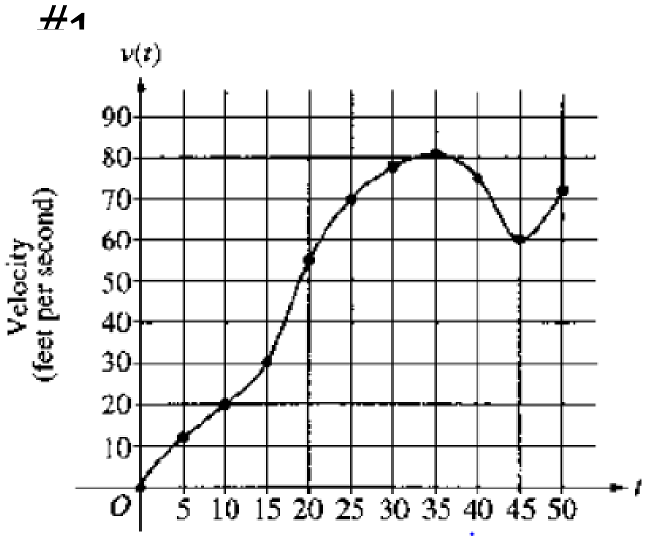
Problems for /

#1



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60

The graph of the velocity v(t), in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for v(t), at 5-second intervals of time t, is shown to the right of the graph.

a. During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

b. Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

c. Find one approximation for the acceleration of the car in ft/sec^2, at t = 40. Show the computations you used to arrive at your answer.

d. Approximate the integral from 0 to 50 of v(t) dt with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

#2

Let f be the function with f(1) = 4 such that for all points (x,y) on the graph of the slope is given by $\frac{3x^2+1}{2y}$.

a. Find the slope of the graph of f at the point where x = 1.

b. Write an equation for the line tangent to the graph of f at x = 1 and use it to approximate f(1.2).

c. Find f(x) by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the initial condition f(1) = 4.

d. Use your solution from part (c) to find f(1.2).

#3

Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

a. Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

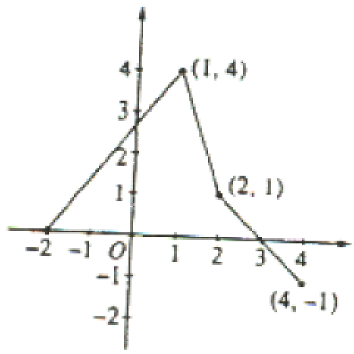
b. Write an equation of each horizontal tangent line to the curve.

c. The line through the origin with slope -1 is tangent to the curve at point P. Find the x- and y-coordinates of point P.

#4

The graph of the function f, consisting of three line segments, is given below.
Let

$$g(x) = \int_1^x f(t)\,dt.$$



a. Compute g(4) and g(-2).

b. Find the instantaneous rate of change of g, with respect to x, at x = 1.

c. Find the absolute minimum value of g on the closed interval [-2,4]. Justify your answer.

d. The second derivative of g is not defined at x = 1 and x = 2. How many of these values are x-coordinates of points of inflection of the graph of f? Justify your answer.

Answers

#1

(a) Acceleration is positive on $(0, 35)$ and $(45, 50)$ because the velocity $v(t)$ is increasing on $[0, 35]$ and $[45, 50]$

(b) Avg. Acc. $= \dfrac{v(50) - v(0)}{50 - 0} = \dfrac{72 - 0}{50} = \dfrac{72}{50}$ or 1.44 ft/sec^2

(c) Difference quotient; e.g.

$$\frac{v(45) - v(40)}{5} = \frac{60 - 75}{5} = -3 \text{ ft/sec}^2 \quad \text{or}$$

$$\frac{v(40) - v(35)}{5} = \frac{75 - 81}{5} = -\frac{6}{5} \text{ ft/sec}^2 \quad \text{or}$$

$$\frac{v(45) - v(35)}{10} = \frac{60 - 81}{10} = -\frac{21}{10} \text{ ft/sec}^2$$

–or–

Slope of tangent line, e.g.

through $(35, 90)$ and $(40, 75)$: $\dfrac{90 - 75}{35 - 40} = -3 \text{ ft/sec}^2$

(d) $\displaystyle\int_0^{50} v(t)\,dt$

$\approx 10[v(5) + v(15) + v(25) + v(35) + v(45)]$
$= 10(12 + 30 + 70 + 81 + 60)$
$= 2530$ feet

This integral is the total distance traveled in feet over the time 0 to 50 seconds.

#2

(a) $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y}$

$$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=4}} = \frac{3+1}{2 \cdot 4} = \frac{4}{8} = \frac{1}{2}$$

(b) $y - 4 = \dfrac{1}{2}(x - 1)$

$$f(1.2) - 4 \approx \frac{1}{2}(1.2 - 1)$$

$$f(1.2) \approx 0.1 + 4 = 4.1$$

(c) $2y\,dy = (3x^2 + 1)\,dx$

$$\int 2y\,dy = \int (3x^2 + 1)\,dx$$

$$y^2 = x^3 + x + C$$

$$4^2 = 1 + 1 + C$$

$$14 = C$$

$$y^2 = x^3 + x + 14$$

$y = \sqrt{x^3 + x + 14}$ is branch with point $(1, 4)$

$$f(x) = \sqrt{x^3 + x + 14}$$

(d) $f(1.2) = \sqrt{1.2^3 + 1.2 + 14} \approx 4.114$

#3

(a) $6y^2\frac{dy}{dx} + 6x^2\frac{dy}{dx} + 12xy - 24x + 6\frac{dy}{dx} = 0$

$\frac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$

$\frac{dy}{dx} = \frac{24x - 12xy}{6x^2 + 6y^2 + 6} = \frac{4x - 2xy}{x^2 + y^2 + 1}$

(b) $\frac{dy}{dx} = 0$

$4x - 2xy = 2x(2 - y) = 0$

$x = 0$ or $y = 2$

When $x = 0$, $2y^3 + 6y = 1$; $y = 0.165$

There is no point on the curve with y coordinate of 2.

$y = 0.165$ is the equation of the only horizontal tangent line.

(c) $y = -x$ is equation of the line.

$2(-x)^3 + 6x^2(-x) - 12x^2 + 6(-x) = 1$

$-8x^3 - 12x^2 - 6x - 1 = 0$

$x = -1/2$, $y = 1/2$

–or–

$\frac{dy}{dx} = -1$

$4x - 2xy = -x^2 - y^2 - 1$

$4x + 2x^2 = -x^2 - x^2 - 1$

$4x^2 + 4x + 1 = 0$

$x = -1/2$, $y = 1/2$

#4

(a) $g(4) = \int_1^4 f(t)\,dt = \frac{3}{2} + 1 + \frac{1}{2} - \frac{1}{2} = \frac{5}{2}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\frac{1}{2}(12) = -6$

(b) $g'(1) = f(1) = 4$

(c) g is increasing on $[-2, 3]$ and decreasing on $[3, 4]$.

Therefore, g has absolute minimum at an endpoint of $[-2, 4]$.

Since $g(-2) = -6$ and $g(4) = \frac{5}{2}$,

the absolute minimum value is -6.

(d) One; $x = 1$

On $(-2, 1)$, $g''(x) = f'(x) > 0$

On $(1, 2)$, $g''(x) = f'(x) < 0$

On $(2, 4)$, $g''(x) = f'(x) < 0$

Therefore $(1, g(1))$ is a point of inflection and $(2, g(2))$ is not.