Practice Problems – Week 1

1. $\int \cos(3x)\,dx =$

   (A) $-3\sin(3x)+C$

   (B) $-\frac{1}{3}\sin(3x)+C$

   (C) $\frac{1}{3}\sin(3x)+C$

   (D) $\sin(3x)+C$

   (E) $3\sin(3x)+C$

2. $\lim_{x\to 0}\frac{2x^6+6x^3}{4x^5+3x^3}$ is

   (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) nonexistent

$$f(x)=\begin{cases}x^2-3x+9 & \text{for } x\le 2\\ kx+1 & \text{for } x>2\end{cases}$$

3. The function f is defined above. For what value of k, if any, is f continuous at $x=2$?

   (A) 1

   (B) 2

   (C) 3

   (D) 7

   (E) No value of k will make f continuous at $x=2$.

4. If $f(x)=\cos^3(4x)$, then $f'(x)=$

   (A) $3\cos^2(4x)$

   (B) $-12\cos^2(4x)\sin(4x)$

   (C) $-3\cos^2(4x)\sin(4x)$

   (D) $12\cos^2(4x)\sin(4x)$

   (E) $-4\sin^3(4x)$

5. The function f given by $f(x)=2x^3-3x^2-12x$ has a relative minimum at $x=$

   (A) -1 (B) 0 (C) 2 (D) $\frac{3-\sqrt{105}}{4}$ (E) $\frac{3+\sqrt{105}}{4}$

6. Let f be the function given by $f(x) = (2x-1)^5(x+1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

(A) $y = 21x + 2$

(B) $y = 21x - 19$

(C) $y = 11x - 9$

(D) $y = 10x + 2$

(E) $y = 10x - 8$

7. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

(A) $2e^{\sqrt{x}} + C$

(B) $\frac{1}{2}e^{\sqrt{x}} + C$

(C) $e^{\sqrt{x}} + C$

(D) $2\sqrt{x}\,e^{\sqrt{x}} + C$

(E) $\frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}} + C$

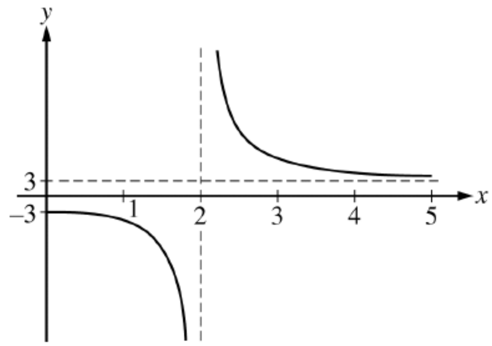
x	0	2	4	6
$f(x)$	4	k	8	12

8. The function f is continuous on the closed interval $[0, 6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)\,dx$ found with 3 subintervals of equal length is 52. What is the value of k ?

(A) 2 (B) 6 (C) 7 (D) 10 (E) 14

9. A particle moves along the x-axis so that at any time $t > 0$, its velocity is given by $v(t) = 4 - 6t^2$. If the particle is at position $x = 7$ at time $t = 1$, what is the position of the particle at time $t = 2$?

(A) -10 (B) -5 (C) -3 (D) 3 (E) 17



10. The function f is given by $f(x) = \frac{ax^2 + 12}{x^2 + b}$. The figure above shows a portion of the graph of f. Which of the following could be the values of the constants a and b ?

(A) $a = -3,\ b = 2$

(B) $a = 2,\ b = -3$

(C) $a = 2,\ b = -2$

(D) $a = 3,\ b = -4$

(E) $a = 3,\ b = 4$

/

11. What is the slope of the line tangent to the graph of $y = \frac{e^{-x}}{x+1}$ at $x = 1$?

(A) $-\frac{1}{e}$ (B) $-\frac{3}{4e}$ (C) $-\frac{1}{4e}$ (D) $\frac{1}{4e}$ (E) $\frac{1}{e}$

12. If $f'(x) = \frac{2}{x}$ and $f(\sqrt{e}) = 5$, then $f(e) =$

(A) 2 (B) ln 25 (C) $5 + \frac{2}{e} - \frac{2}{e^2}$ (D) 6 (E) 25

13. $\int (x^3 + 1)^2 \, dx =$

(A) $\frac{1}{7}x^7 + x + C$

(B) $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$

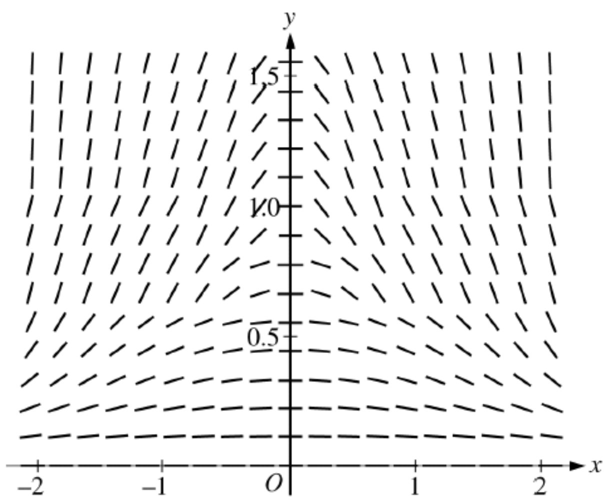
(C) $6x^2(x^3 + 1) + C$

(D) $\frac{1}{3}(x^3 + 1)^3 + C$

(E) $\frac{(x^3 + 1)^3}{9x^2} + C$

14. $\lim_{h \to 0} \frac{e^{(2+h)} - e^2}{h} =$

(A) 0 (B) 1 (C) $2e$ (D) e^2 (E) $2e^2$



15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cos x$

(B) $y = 1 - x^2$

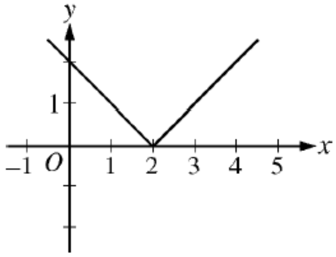
(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

(E) $y = \frac{1}{1 + x^2}$

16. If $f'(x) = |x - 2|$, which of the following could be the graph of $y = f(x)$?

(A)



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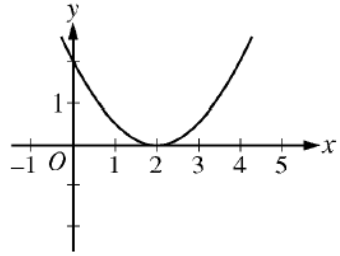

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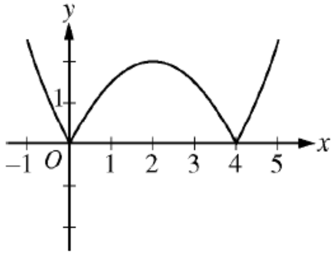

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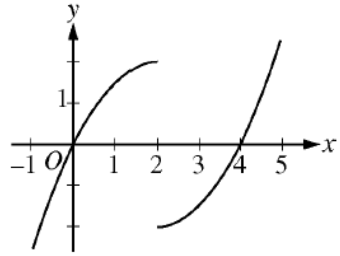

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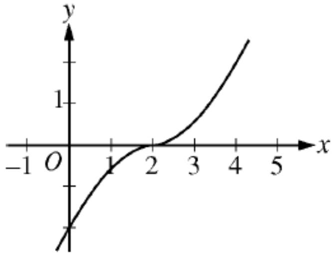


18. For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

(A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64

19. Let f be the function given by $f(x) = x^3 - 6x^2$. The graph of f is concave up when

(A) $x > 2$

(B) $x < 2$

(C) $0 < x < 4$

(D) $x < 0$ or $x > 4$ only

(E) $x > 6$ only

20. If $g(x) = x^2 - 3x + 4$ and $f(x) = g'(x)$, then $\int_1^3 f(x)\,dx =$

(A) $-\frac{14}{3}$ (B) -2 (C) 2 (D) 4 (E) $\frac{14}{3}$

22. If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x = 2$

II. A relative minimum at $x = 3$

III. A relative maximum at $x = 4$

(A) I only

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III

24. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

25. If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$

(A) $-\frac{7}{11}$ (B) $-\frac{7}{13}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{14}$ (E) 7

26. For $x > 0$, f is a function such that $f'(x) = \frac{\ln x}{x}$ and $f''(x) = \frac{1 - \ln x}{x^2}$. Which of the following is true?

(A) f is decreasing for $x > 1$, and the graph of f is concave down for $x > e$.

(B) f is decreasing for $x > 1$, and the graph of f is concave up for $x > e$.

(C) f is increasing for $x > 1$, and the graph of f is concave down for $x > e$.

(D) f is increasing for $x > 1$, and the graph of f is concave up for $x > e$.

(E) f is increasing for $0 < x < e$, and the graph of f is concave down for $0 < x < e^{3/2}$.

27. If f is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t}\, dt$, then $f'(2) =$

(A) 0 (B) $\frac{7}{2\sqrt{12}}$ (C) $\sqrt{2}$ (D) $\sqrt{12}$ (E) $2\sqrt{12}$

28. If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1 + 25x^2}$

(B) $\frac{5}{1 + 25x^2}$

(C) $\frac{-5}{\sqrt{1 - 25x^2}}$

(D) $\frac{1}{\sqrt{1 - 25x^2}}$

(E) $\frac{5}{\sqrt{1 - 25x^2}}$

77. If $\int_0^3 f(x)\,dx = 6$ and $\int_3^5 f(x)\,dx = 4$, then $\int_0^5 (3 + 2f(x))\,dx =$

(A) 10 (B) 20 (C) 23 (D) 35 (E) 50

78. For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$?

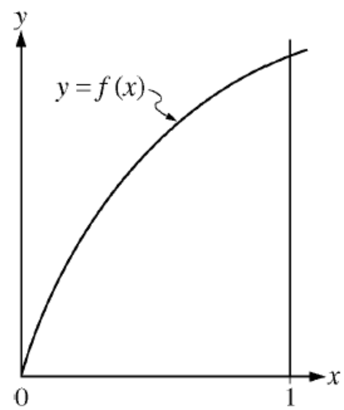
(A) The change in temperature during the first day

(B) The change in temperature during the 24th hour

(C) The average rate at which the temperature changed during the 24th hour

(D) The rate at which the temperature is changing during the first day

(E) The rate at which the temperature is changing at the end of the 24th hour



80. A left Riemann sum, a right Riemann sum, and a trapezoidal sum are used to approximate the value of $\int_0^1 f(x)\,dx$, each using the same number of subintervals. The graph of the function f is shown in the figure above. Which of the sums give an underestimate of the value of $\int_0^1 f(x)\,dx$?

I. Left sum
II. Right sum
III. Trapezoidal sum

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

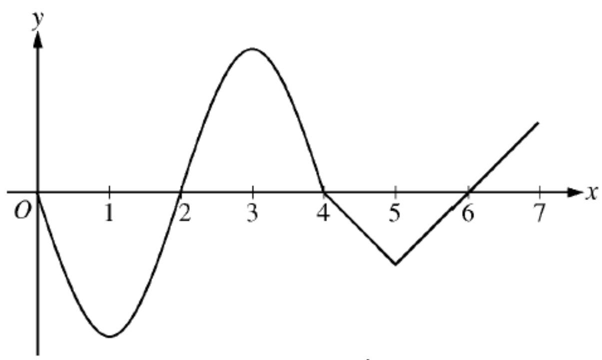
82. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

(A) There is a number c in the open interval (a, b) such that $f(c) = 0$.
(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.
(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all x in $[a, b]$.
(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$.
(E) There is a number c in the open interval (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

83. The function f is differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \le x \le 5$. Which of the following could be the value of $f'(3)$?

(A) 20 (B) 27.5 (C) 29 (D) 30 (E) 30.5



Graph of f'

84. The graph of f', the derivative of the function f, is shown above. On which of the following intervals is f decreasing?

(A) [2, 4] only

(B) [3, 5] only

(C) [0, 1] and [3, 5]

(D) [2, 4] and [6, 7]

(E) [0, 2] and [4, 6]

x	3	4	5	6	7
$f(x)$	20	17	12	16	20

86. The function f is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

I. The minimum value of f on $[3, 7]$ is 12.

II. There exists c, for $3 < c < 7$, such that $f'(c) = 0$.

III. $f'(x) > 0$ for $5 < x < 7$.

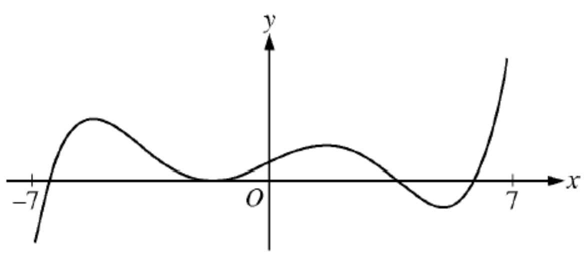
(A) I only

(B) II only

(C) III only

(D) I and III only

(E) I, II, and III



Graph of f'

87. The figure above shows the graph of f', the derivative of the function f, on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?

(A) One (B) Two (C) Three (D) Four (E) Five

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	–2	–3	4

89. The table above gives values of the differentiable functions f and g and their derivatives at $x = 1$. If $h(x) = (2f(x) + 3)(1 + g(x))$, then $h'(1) =$

(A) −28 (B) −16 (C) 40 (D) 44 (E) 47

90. The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

No.	Correct Answer
1	C
2	D
3	C
4	B
5	C
6	B
7	A
8	D
9	C
10	D
11	B
12	D
13	B
14	D
15	E
16	E
17	D
18	D
19	A
20	C
21	C
22	A
23	B
24	D
25	A
26	C
27	E
28	E

No.	Correct Answer
76	C
77	D
78	E
79	A
80	D
81	B
82	C
83	D
84	E
85	D
86	B
87	A
88	C
89	D
90	E
91	D
92	C